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A Deteriorating Inventory Model with Quadratic Time Dependent Demand under Partial Backlogging

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Abstract: **In this research, we study deteriorating inventory replenishment policy with quadratic trend in demand and shortages over a fixed planning period and we obtain the analytic solution by minimizing the total inventory.**

Keywords: **Inventory model, deteriorating items, quadratic, time-dependent demand and partial backlogging.**

1. INTRODUCTION

Inventory is define as the stock of goods or other economic resources that are kept, stored, accumulated or reserved in order to guarantee even and efficient running of business activities. The cause of deterioration cannot be neglected in inventory management model.

Deterioration is defined as spoilage, damage, decay, or change that prevents the material from being used for its initial purpose. Example of items in which deterioration can occurs are drugs, radioactive substance, food items, pharmaceuticals, photographic firm, electronic component and fashionable cloths.

The first economic order quantity was developed by Harris (1915).And is extended by Wilson (1934) which give the general formula to obtain economic order quantity. Ghare and Schrader (1963) designed an inventory for an exponentially decay inventory. Convert and Philip (1973) enrich Ghare and Schrader's model for two parameter Weilbull distribution deterioration. Goyal (1985) create an economic order quantity permissible delay in payment by neglecting the difference between the selling price and purchase cost and finalized that the economic the economic replenishment interval and order quantity generally increases marginally under permissible delay in payment .Dave (1985) make a correction in Goyal's model by assuming that the selling price is necessarily higher than its purchase price. Aggarwal and Jaggi (1995) expand Goyal's model by considering deteriorating items. Jamal etal (1997) also improve Goyal's model by allowing shortages and deterioration. Chang and Dye (2001) enlarge Jamal et al to allow time varying deterioration rate and backlogging rate to be inversely proportional to the waiting time. Teng (2002) established economic order quantity model with zero deteriorating rate. Chang et al (2003) expanded Teng's model by considering a permissible delay to the purchase if the other quantity is greater than or equal to a predetermined quantity. More recently, Roy (2008) established economic order quantity with time proportion deteriorating rating rate, Demand rate as function of selling price and time dependent holding cost. Moshra et al (2013) improve Roy's model by designing an inventory model for deteriorating items with demand rate and holding cost as increasing linear function of time with shortages (2014) enrich Mishra et al model by considering linear time dependent demand and develop an inventory deteriorating items where deteriorating rate and holding cost are constant with shortage and partial backlogged. In this work Fortity Milu and Shrutirekha's model by considering quadratic time dependent demand rate.

This work has been arranged into various important sections which includes;

- Introduction
- Assumption and notations
- Mathematical formulation and
- Conclusion.

2. NOTATIONS AND ASSUMPTIONS

We need the following notations for developing mathematical model;

1. The demand rate is time dependent and quadratic ;

$$D(t) = a_1 + a_2 t + a_3 t^2$$

Where $a_1, a_2, a_3 > 0$ constants

2. The replenishment is instantaneous
3. $I(t)$ is the level of inventory at time t , $0 \le t \le T$
4. T is the length of the cycle
5. θ is the constant deteriorating rate, $0 < \theta < 1$
6. t_1 is the time when the inventory level reaches zero
7. ϕ is the ordering quantity per circle.
8. A_0 is the fixed ordering per order.
9. C_d is the cost of each deterioration.
10. C_h is the inventory holding cost per unit time.
11. C_s is the shortage cost per unit of time.
12. M is the maximum inventory level for the ordering cycle such that $M = I(0)$.
13. $A_T(t_1)$ is the average total cost per unit time under condition $t_1 \le T$.
14. $\bar{t}_1$ is the optimal point.

3. MATHEMATICAL FORMULATION

Here we consider the deteriorating inventory model with quadratic time dependent demand rate. Replenishment occurs at t = 0, when the inventory level attain it maximum from t = 0 to t_1, the inventory level reduces due to demand and deterioration. At t_1 the inventory level achieves zero, then shortage is allowed to occur during the time interval (t_1, T) is completely back logged. The total number of back logged items is replaced by the next replenishment .According to the notations and assumptions mentioned above, the behavior of inventory system at any time can be described by the following differential equations.

$$\frac{dI_1(t)}{dt} = -D(t) - \theta I_1(t), \quad 0 \le t \le t_1 \qquad \cdots(1)$$

$$\frac{dI_2(t)}{dt} = -D(t), \qquad t_1 \le t \le T \qquad \cdots(2)$$

Equation (1) is a linear first order equation, hence

$$\left(I_1(t)e^{\theta t}\right)^{\mathrm{I}} = -\left(a_1 + a_2 t + a_3 t^2\right)e^{\theta t}$$

On integration, the above equation gives

$$I_1(t)e^{\theta t} = -\left[\left(a_1 + a_2 t + a_3 t^2\right)\frac{e^{\theta t}}{\theta} - \left(a_2 + 2a_3 t\right)\frac{e^{\theta t}}{\theta^2} + \left(2a_3\right)\frac{e^{\theta t}}{\theta^3}\right] + C_1$$

Where C_1 is an integral constant using the given boundary conditions, solution of the above differential equation is given by

$$I_1(t) = \left[\frac{a_1 + a_2 t_1 + a_3 t_1^2}{\theta} - \frac{a_2 + 2a_3 t_1}{\theta^2} + \frac{2a_3}{\theta^3}\right]e^{\theta(t_1 - t)} - \frac{a_1 + a_2 t + a_3 t^2}{\theta} + \frac{a_2 + 2a_3 t}{\theta^2} - \frac{2a_3}{\theta^3}$$

Since $I_1(0) = M$, we get

$$M = I_1(0) = \left(\frac{a_1}{\theta} - \frac{a_2}{\theta^2} + \frac{2a_3}{\theta^3}\right)\left(e^{\theta t_1} - 1\right) + \left(\frac{a_2 t_1}{\theta} - \frac{a_3 t_1^2}{\theta} + \frac{2a_3 t_1}{\theta^2}\right)e^{\theta t_1}$$

Solving equation (2) and its solution is given by

$$I_2(t) = a_1(t_1 - T) + \frac{a_2}{2}\left(t_1^2 - T^2\right) + \frac{a_3}{3}\left(t_1^3 - T^3\right)$$

In the circle time the number of deteriorating units (NDU) is given by

$$NDU = M - \int_0^{t_1} D(t)\,dt$$

$$= M - \int_0^{t_1}\left(a_1 + a_2 t + a_3 t^2\right)dt$$

$$= \left(\frac{a_1}{\theta} - \frac{a_2}{\theta^2} + \frac{2a_3}{\theta^3}\right)\left(e^{\theta t_1} - 1\right) + \left(\frac{a_2 t_1}{\theta} - \frac{a_3 t_1^2}{\theta} + \frac{2a_3 t_1}{\theta^2}\right)e^{\theta t_1} - \left(a_1 t + \frac{a_2 t^2}{2} + \frac{a_3 t^3}{3}\right)$$

The total number of inventory carried during the interval $[0, t_1]$, is given by

$$I_C = \int_0^{t_1} I_1(t)dt$$

$$= \int_0^{t_1}\left[\frac{a_1 + a_2 t_1 + a_3 t_1^2}{\theta} - \frac{a_2 + 2a_3 t_1}{\theta^2} + \frac{2a_3}{\theta^3}\right]e^{\theta(t_1 - t)} - \frac{a_1 + a_2 t + a_3 t^2}{\theta} + \frac{a_2 + 2a_3 t}{\theta^2} - \frac{2a_3}{\theta^3}$$

$$= \left[\frac{a_1 + a_2 t_1 + a_3 t_1^2}{\theta} - \frac{a_2 + 2a_3 t_1}{\theta^2} + \frac{2a_3}{\theta^3}\right]\left(e^{\theta t_1} - 1\right) - \frac{a_1\theta^2 - a_2 + 2a_3}{\theta^3}t_1 - \frac{a_2\theta^2 + 4a_3}{2\theta^3}t_1^2 - \frac{a_3}{3\theta}t_1^3$$

The total shortage during the interval $[t_1, T]$, say T_s, is

$$T_s = -\int_{t_1}^{T} I_2(t)\,dt$$

$$= -\int_{t_1}^{T} \left[a_1(t_1 - T) + \frac{a_2}{2}(t_1^2 - T^2) + \frac{a_3}{3}(t_1^3 - T^3) \right] dt$$

$$= \frac{a_1}{2}(T^2 + t_1^2 - 2t_1T) + \frac{a_2}{6}(T^3 + 2t_1^3 - 3t_1^2T) + \frac{a_3}{12}(T^4 + 3t_1^4 - 4t_1^4T) \quad ...(8)$$

Then, the average total cost per unit time under the condition $t_1 \le T$ can be given by

$$A_T(t_1) = \frac{1}{T}[A_0 + C_d.NDU + C_h I_c + C_s T_s] \quad ...(9)$$

The first order derivative of $A_T(t_1)$ with respect to t_1 is as follows:

$$\frac{dA_T(t_1)}{dt_1} = \frac{1}{T}\left[\left(C_d + \frac{C_n}{\theta} \right)(e^{\theta t_1} - 1) + C_s(t_1 - T) \right](a_1 + a_2t_1 + a_3t_1^2) \quad ...(10)$$

The necessary condition for $A_T(t_1)$ in (9) to be minimized is

$$\frac{dA_T(t_1)}{dt_1} = 0, \text{ That is}$$

$$\left[\left(C_d + \frac{C_h}{\theta} \right)(e^{\theta t_1} - 1) + C_s(t_1 - T) \right](a_1 + a_2t_1 + a_3t_1^2) = 0 \quad ...(11)$$

Let $f(t_1) = \left[\left(C_d + \frac{C_h}{\theta} \right)(e^{\theta t_1} - 1) + C_s(t_1 - T) \right]$

Since $f(0) = -C_sT < 0$, $f(T) = \left(C_d + \frac{C_h}{\theta} \right)(e^{\theta T} - 1) > 0$

For $e^{\theta T} > 1$

And $f^{I}(t_1) = (\theta C_d + C_h)e^{\theta t_1} + C_s > 0,$ It implies that $f(t_1)$ is strictly monotonic Increasing function and equation (11) has a unique solution at $\bar{t}_1$, for $\bar{t}_1 \in (0, T)$.

Therefore, the deteriorating inventory model under the condition $0 < t_1 \le T$, $A_T(t_1)$ obtain its maximum at $t_1 = \bar{t}_1$, where $f(\bar{t}_1) = 0$ if $\bar{t}_1 < T$.

4. CONCLUSION

In this work, we study inventory model for deteriorating items with quadratic time dependent demand rate. We proposed an inventory replenishment policy for this type of inventory model. The proposed model can further be enriched by taking more realistic assumptions such as probabilistic demand rate.

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